## ELASTICITY THEORY SOLUTION OF THE PROBLEM ON PLANE BENDING OF A NARROW LAYERED CANTILEVER BEAM BY LOADS AT ITS FREE END

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An exact elasticity theory solution for the problem on plane bending of a narrow layered composite cantilever beam by tangential and normal loads distributed on its free end is presented. Components of the stress-strain state are found for the whole layers package by directly integrating differential equations of the plane elasticity theory problem by using an analytic representation of piecewise constant functions of the mechanical characteristics of layer materials. The continuous solution obtained is realized for a four-layer beam with account of kinematic boundary conditions simulating the rigid fixation of its one end. The solution obtained allows one to predict the strength and stiffness of composite cantilever beams and to construct applied analytical solutions for various problems on the elastic bending of layered beams.

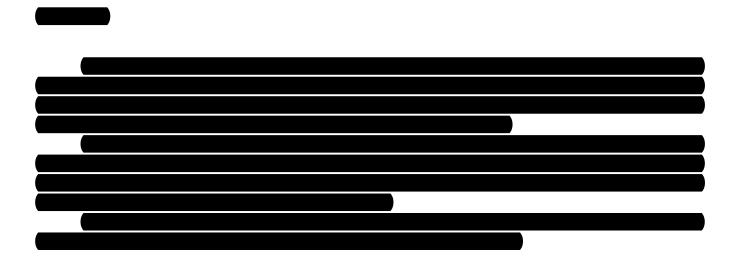
## Introduction



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