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ANALYTICAL SOLUTION TO THE PLANE BENDING TASK OF THE MULTILAYER BEAM WITH A CIRCULAR AXIS UNDER NORMAL UNIFORM LOADING

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Circular bars are widespread elements in building and mechanical engineering. They allow one to enhance the strength of the multilayer composite materials along with their weight reduction. However, their analytical modeling is developed rather poorly, which creates additional obstacles in their efficient design. The paper presents the results of the exact analytical solution to the plane bending task of elasticity theory for the multilayer beam with a circular axis under normal uniform loading on its longitudinal surfaces considering the cylindrical orthotropy of its layers' materials. The problem has been solved using the methods of linear elasticity theory based on the continual approach where the circular bar is considered as a solid with the elastic characteristics varying in the radial direction. The solution for the shear stresses is assumed using the known distribution in the cross section. Therefore, the solution to the problem is reduced to the determination of two unknown functions of stress distribution throughout the section height. The system of linear differential equations has been obtained for the determination of the sought functions and unknown functions of integration in the process of the solution to the equations of plane theory considering the variable elastic characteristics. General analytical solutions for uniform and discontinuous-nonuniform layers from cylindrically orthotropic and isotropic materials have been developed. The influence of transverse shear deformation and compression on the characteristics of the stress-strain state of a cantilever three-layer beam with a semicircular axis is theoretically investigated. The obtained relations allow us to predict the stress-strain state of circular multilayer beams, as well as to build applied methods for solving problems of strength, rigidity, and optimal design of such structural elements.

Keywords: multilayer beam, circular axis, plane bending, stress, strains, displacement.

Introduction. Circular beams are widespread elements of the building and machine-building structures both in the form of individual parts and reinforcing elements of thin-walled shells. Replacing the homogeneous material of such elements with an inhomogeneous multilayer composite structure allows, with appropriate selection of the layers' materials, to increase their strength together with a significant weight loss. However, the insufficient development of analytical theories of bending of multilayer circular beams creates additional obstacles for their efficient design.

In many cases, the model of a linear-elastic body is used as the first approximation in solving the problems of composite mechanics [1] since fibrous composites have an almost linear relationship between stresses and strains up to failure [2]. This indicates the relevance of the application of the methods of elasticity theory of an inhomogeneous body for the study of the stress-strain state (SSS) of composite elements, as well as, under certain conditions, their fracture mechanics.

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CONCLUSIONS

1. The plane bending problem of a circular multilayer beam under the action of uniformly distributed loads acting on the longitudinal cylindrical surfaces with perfectly rigid connection and elastic work of cylindrically orthotropic layers was solved using the elasticity theory approach. The solution was analytically reduced to the definition of two continuous functions of the radial coordinate, describing the components of the distribution of normal transverse and tangential stresses in the beam cross section.

2. A system of relations and conditions was developed, which allows one to derive all unknown functions and constants, as well as the distribution of stresses, strains, and displacements for the entire set of cylindrically orthotropic homogeneous or continuously heterogeneous layers of the beam.

3. The obtained solution is an exact solution to the plane problem of the elasticity theory, provided that the self-balanced normal load is distributed at the beam end, which practically does not affect the SSS in far beam sections, according to the local load action principle.

4. Based on the test results obtained, the hypothesis of plane sections was shown to slightly affect the distribution of transverse normal and tangential stresses, but significantly influence that of longitudinal normal stresses.

5. Transverse shear deformations and cross-sectional reduction significantly affect the distribution and magnitude of displacements. However, no obvious warping of cross sections is observed, in contrast to straight beams.

6. The obtained solution is instrumental in the accurate SSS prediction of circular multilayer beams and can be used to develop applied methods for solving problems of strength, rigidity, optimal design, and experimental determination of the elastic characteristics of individual layers of composite beams.

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