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SOLVING THE PROBLEM OF ELASTIC BENDING OF A LAYERED CANTILEVER UNDER A NORMAL LOAD LINEARLY DISTRIBUTED OVER LONGITUDINAL FACES

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We present the exact solution to the problem of elastic plane bending of a layered composite beam of small width under normal loads linearly distributed over the longitudinal faces. The constitutive equations for the strain–stress state are obtained by directly solving a plane problem of elasticity taking into account the orthotropy of the elastic properties of layer materials and their piecewise-constant variation along the cross-sectional height. The obtained analytical solution describes the distribution of stresses and displacements over the whole layered bar and allows performing strength and rigidity analyses as well as finding the solutions to different applied problems on plane bending of composite beams.

Keywords: beam, composite, layer, bending, stress, deformation, displacement

Introduction. Composite structural elements have a good combination of physical and mechanical properties. These properties can be achieved by combining several dissimilar materials. However, such a technique creates anisotropy of the properties and discrete inhomogeneity of the material. This major problem does not allow us to use traditional methods that are based on the homogeneity and isotropy of material properties.

Plane transverse bending is a very common type of deformation for beam elements of engineering structures. However, the analytical theory of deformation of composite beams is the least developed, in comparison with plates and shells.

Scientific studies on theoretical problems of modeling the bending of composite beams widely use non-classical or "refined" models. Such models are mainly based on the hypotheses of the "classical" bending model taking into account the inhomogeneity of materials and transverse compression and shear. The hypothesis are refined either once in the initial modeling [12 – 14] or at each step of the iterative process [7, 11]. Such models are quite universal and, in the majority of practical cases, provide acceptable accuracy of the stress–strain state (SSS).

Obtaining accurate solutions to the equations of elasticity, taking into account the inhomogeneity of materials, is a more accurate and sophisticated method for modeling the bending of composite beams. Muskhelishvili [3, 4], Lekhnitskii [1, 2], and Pagano [10] found some solutions to such problems using some simplifications. Application of these solutions are complicated by the fact that they are general and require additional theoretical studies to be used in particular cases. They also allow us to describe only continuous inhomogeneity and boundary conditions of a particular type. However, such solutions more accurately describe the SSS of an inhomogeneous beam, and several solutions for different loads make it possible to solve more complex problems. Therefore, the development of this method of modeling the bending of composite beams is important and relevant.

The exact solution of the problem of the elastic bending of a narrow cantilever was obtained in [8] for a load applied at the end and in [9] for uniformly distributed normal loads applied to the longitudinal sides. Combining these solutions allows us to find applied solutions to many problems of the deformation of layered beams under different combinations of typical loads.

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(2.13), (2.40), (2.41), (2.43)–(2.45) constitute the exact solution to the problem of the theory of elasticity if there is tangent load (2.31) with zero resultant on the free end. The constructed solution allows us to take into account an arbitrary number and order of alternation of layers, as well as the orthotropy of the elastic characteristics of their materials.

The theoretical relations have been tested in determining the SSS of a three-layer short cantilever. The results show a significant effect of the compliance of the layer material to transverse shear and compression on the distribution of normal stresses σ_x. The transverse compression stresses σ_z and tangential stresses τ_{xz} can be determined with practically sufficient accuracy using the plane section hypothesis.

The compliance of the beam material to the shear and compression also significantly affects the distribution and maximum values of longitudinal displacements *u* and, especially, transverse displacements *w*.

The obtained solution can be used to predict the strength and stiffness of multilayer cantilever beams under plane bending, as well as to develop applied methods for designing such structural elements.

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